Fractional order optimal intensity measures for probabilistic seismic demand modeling of extended pile-shaft-supported bridges in liquefiable and laterally spreading ground

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A R T I C L E   I N F O

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A B S T R A C T

In performance-based earthquake engineering, probabilistic seismic demand models of structures are essential components that provide probabilistic estimates of earthquake-induced demands as a function of a variable(s) called the ground motion intensity measure (IM). Uncertainties in these models are often dependent on the IM used. Extending from traditional integer order IMs, this study assesses the performance of fractional order (FO) IMs on the probabilistic seismic demand modeling of extended pile-shaft supported bridges sited in liquefiable and laterally spreading ground. Uncertainties in structural and geotechnical material properties as well as geometric parameters of the bridges are considered in finite element models to achieve comprehensive scenarios. The FO IMs considered include peak ground response (PGR), cumulative absolute response (CAR) and its modified version (CAR_m), spectral acceleration at 2.0 s for a fractionally damped single degree of freedom (SDF) system (S_ad-20a) and for a conventional SDF system with fractional response (S_pr-20a), spectrum intensity for a fractionally damped SDF system (SI_d), as well as for a conventional SDF system with fractional response (SI_s). Metrics such as efficiency, practicality, proficiency and sufficiency are measured to assess the optimal IM with respect to different demand parameters. Results show the advantages of FO IMs as they increase confidence in demand models compared to traditional integer order IMs. In particular, the proposed fractional spectrum intensities (SI_d and SI_s) with their optimal values produce significant improvements in practicality, efficiency and proficiency, while maintaining sufficiency. Therefore, FO IMs can provide more reliable demand models for probabilistic seismic demand analysis of extended pile-shaft supported bridges in liquefiable and laterally spreading ground.

1. Introduction

In performance-based earthquake engineering, the seismic behavior of a structure, such as a bridge, should be predicted and evaluated with sufficient confidence. To this end, a probabilistic seismic demand model (PSDM) with low uncertainty constitutes a key component in the development of fragility models and conducting risk analysis for risk-informed decision making. Traditionally, a PSDM provides a relationship between a structural demand parameter (e.g. column drift, bearing deformation, section curvature, etc.) and a ground motion intensity measure (IM) [1]. In other words, it is a conditional expression of the probability that a structural component experiences a demand for a given IM level, indicating the importance of the IM as a conditional variable in the PSDM. Therefore, optimal selection of the IM is an essential task that helps to increase the confidence in PSDMs and subsequent risk estimates used in decision making.

Several studies have reported the selection of optimal IMs for different structures. While the majority of the related works address building structures [2–7], several past studies have also explored IMs for bridge systems. Padgett et al. [8] reported that peak ground acceleration (PGA) is a prime IM for portfolios of highway bridges in the Central and Eastern United States based on comprehensive assessment of various characteristics of an optimal IM, such as efficiency, practicality, proficiency, sufficiency and hazard computability, which will be briefly described later in this paper. Mackie and Stojadinović [9] found that spectral acceleration and displacement at the fundamental period...
(\(S_{a,20}\) and \(S_{d,20}\), respectively) are appropriate IMs for PSDMs of California highway bridges. To improve accuracy and reduce bias in PSDMs, Baker and Cornell [10] proposed vector-valued IMs comprised of two or more IMs (e.g., \(S_{a}\) at fundamental and higher order modes of vibration). Recently, Zelaschi et al. [11] studied optimal IMs for PSDMs of Italian RC bridge portfolios and revealed that peak ground velocity (PGV), \(S_{a,20}\), and Fajjar Index (\(I_{f}\)) are the optimal scalar IMs. They also found that the studied vector-valued IMs involving these optimal scalar IMs did not show significant improvements as compared to the scalar ones. Feng et al. [12] identified optimal IMs for PSDMs of short-to-medium span curved concrete bridges considering the impact of seismic excitation direction. They found that spectral acceleration at 1.0 s is the optimal IM regardless of the seismic excitation direction. Du et al. [13] recently proposed posteriori optimal IMs that contain parameters with optimal predefined values for PSDMs of single-degree-of-freedom (SDF) systems. They demonstrated that the spectral acceleration with optimal predefined period and damping ratio is the optimal IM. To date, most researchers have adopted acceleration-related IMs such as PGA and \(S_{a,7n}\) in predicting PSDMs and subsequent fragility of highway bridges [14–16]. It should be noted that these studies often used fixed-based or six-spring linear connections for foundation simulations; for these cases, acceleration-related IMs may be reasonable optimal choices as soil behavior has slight or no effects on structure responses.

As to more complex geotechnical conditions such as soil-foundation-structure interactions (SFSI) in the presence of liquefaction, studies on the selection of optimal IMs are less common. Kostadinov and Yamazaki [17] investigated the dynamic response of liquefiable soil embankments and concluded that the occurrence of liquefaction is well correlated with velocity-based IMs such as peak ground velocity (PGV). This trend was verified in wharf structures by Shafeezezadeh et al. [18–20]. Kramer and Mitchell [21] found that a modified version of cumulative absolute velocity (CAV), \(C_{AV}\), exhibits strong correlations with the buildup of excess pore water pressures in soils that represents seismic liquefaction hazard. Bradley et al. [22] reported that Housner spectrum intensity (HI) is the prime IM for the seismic response prediction of pile foundations in liquefiable acyclic ground based on its metrics of efficiency and sufficiency. Wang et al. [23] found that PGV and velocity spectrum intensity (VSI) are optimal IMs for PSDMs of a multi-span continuous steel girder bridge in acyclic liquefiable ground. Karimi and Dashiti [24] studied optimal IMs for estimates of the seismic performance of shallow-founded structures on acyclic liquefiable ground and identified that CAV and \(C_{AV}\) are optimal IMs for permanent settlement, while PGV is the optimal one for rocking drift ratio. Recently, Wang et al. [25] explored the optimal IMs for PSDMs of bridges in liquefiable and laterally spreading ground and found that HI, PGV, CAV, \(C_{AV}\), and spectral acceleration at 2.0 s (\(S_{a,20}\)) are appropriate IMs. In general, although velocity-related and spectrum-based IMs appear to be reasonable choices for probabilistic seismic demand modeling of pile-supported structures that are subject to significant SFSI, further studies are required to verify this inference.

Nevertheless, the abovementioned traditional IMs have two key limitations. One is the fact that they are based on the premise that the seismic behavior of a structure can only be categorized into discrete acceleration, velocity, or displacement sensitive domains. These IMs are obtained through integer order derivatives or integrals of ground motions, e.g., PGA, PGV and peak ground displacement (PGD), or \(S_{a,7n}\), spectral velocity at structural fundamental period (\(S_{a,7n}\)) and \(S_{d,7n}\). However, seismic responses of structures with complex geotechnical conditions that involve a wide range of nonlinear dynamic behaviors may not be properly classified as acceleration, velocity, or displacement sensitive. Therefore, response estimation using conventional IMs such as PGA, PGV, \(S_{a,7n}\), may not provide the optimal model. Another physical limitation of the conventional IMs is the fact that soil-structure systems under earthquakes may exhibit fractional order (FO) features that arise from sources such as seismic wave propagations in geologic media [26–28] and radiation damping in SFSI [29,30]. Traditional IMs derived from integer order derivatives or integrals of ground motions or structure responses with classical forms of damping (proportional to velocity of the system) are not able to fully represent such fractional properties. In this regard, Shafeezezadeh et al. [31] introduced the application of FO calculus in forming novel IMs; that is releasing the constraints of integer orders in the conventional IMs such as PGA, PGV, PGD and \(S_{a,7n}\), and extending them into FO IMs such as peak ground fractional order responses (PGF\(_{\alpha}\), where the subscript \(\alpha\) denotes the fractional order). The superiority of these FO IMs against traditional ones were verified through probabilistic seismic demand modeling of a portfolio of highway bridges in nonliquefiable soils in California. In order for PGF\(_{\alpha}\) to be useful for probabilistic seismic risk analyses, Kale et al. [32] recently proposed a ground motion prediction equation for PGF\(_{\alpha}\) for active shallow crustal regions. In addition to above relevant applications, FO calculus has been applied in fields such as geotechnical and structural earthquake engineering, for instance, to characterize viscoelastic behavior of soils under seismic excitations [33–35], and to identify responses of base-isolated structures [36,37] or structures with dampers [38]. In addition, capabilities of FO models for the damping of viscoelastic materials have been verified by experimental and analytical studies [39,40]. Therefore, for systems such as bridges in liquefiable and laterally spreading ground where viscoelastic behavior of soils and extreme SFSI take place, it is reasonable to speculate that the traditional IM-based PSDMs may not yield optimal results in terms of confidence in seismic demand estimates. Furthermore, to improve the viability and confidence in using FO IMs, it is important to identify whether these IMs consistently outperform their traditional counterparts for different geometric configurations and material properties of soil-structure systems; and if the optimal fractional order, \(\alpha\), is stable for different demand parameters under various scenarios. Also, opportunities exist to extend the set of fractional order IMs that have been recently considered in the literature. These issues provide the motivation for the present study.

This study aims to evaluate the performance of FO IMs compared with traditional IMs and identify the optimal fractional order for PSDMs of bridges in liquefiable and laterally spreading ground where extreme SFSI takes place. Characteristics of an optimal IM are introduced briefly, followed by a description of the studied FO and traditional IMs, including introduction of newly proposed definitions of FO IMs. Then, adopted bridge models in OpenSees [41] considering geotechnical and structural material uncertainties and geometric parameters are described. Following that section, optimal FO IMs for PSDMs of the bridge models in liquefiable and laterally spreading ground are discussed in terms of their ability to predict various demand parameters.

2. PSDM and criteria of an optimal IM

The pioneering work by Cornell et al. [42] indicated that conditional seismic demands can be modeled using a lognormal distribution, as expressed in Eq. (1):

\[
P[D \geq d | IM] = 1 - \Phi \left( \frac{\ln(d) - \ln(S_d)}{\hat{\beta}_{d(IM)}} \right)
\]

(1)

where \(\Phi(\cdot)\) is the standard normal cumulative distribution function, \(d\) is the seismic demand, \(S_d\) is the median value of the demand in terms of an IM, and \(\hat{\beta}_{d(IM)}\) is the logarithmic standard deviation of the demand conditioned on the IM. The relationship between the demand and IM is expressed in the power form given in Eq. (2):

\[
S_d = a \cdot IM^b
\]

(2)

where constant \(a\) and \(b\) are regression parameters. Eq. (2) can be further transformed into the lognormal space, as displayed in Eq. (3):

\[
\ln(S_d) = \ln(a) + b \cdot \ln(IM)
\]

(3)

where \(\ln(a)\) is the vertical intercept and the constant \(b\) is the slope. The
regressions are developed by performing nonlinear time history ana-
lyses on the adopted bridge models using a suite of \( N \) ground motions. The obtained \( N \) demand quantities \( \{d_i, i = 1, 2, \ldots, N\} \) are then plotted against the IM to estimate the regression parameters including \( a, b \), the coefficient of determination \( (R^2) \) as well as the dispersion term, \( \hat{\beta}_{D|IM} \), given in Eq. (4):

\[
\hat{\beta}_{D|IM} \equiv \left[ \frac{\sum_i (\ln(d_i) - \ln(S_0))^2}{N - 2} \right]^{1/2}
\]  

(4)

It is worth noting that the assumptions of power-law formulation in Eq. (2) and constant dispersion in Eq. (4) are not the only possible models to express seismic demands as a function of an IM. However, it has been widely used for fragility assessments in highway bridges [14].

Based on the formulation of the PSDM and its statistical properties, five commonly used measures have been presented in the literature, including efficiency, sufficiency, hazard computability [43], practicality [9] and proficiency [8]. For conciseness, Table 1 summarizes the defini-
tions of these measures. Further explanation on each measure can be

Table 1

<table>
<thead>
<tr>
<th>Measure</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Efficiency</td>
<td>Dispersion, ( \beta_{D</td>
</tr>
<tr>
<td>Practicability</td>
<td>Slope, ( b )</td>
</tr>
<tr>
<td>Proficiency</td>
<td>Modified dispersion, ( \zeta = \beta_{D</td>
</tr>
<tr>
<td>Sufficiency</td>
<td>( p )-values, ( M_p \text{-value} ) and ( R_p \text{-value} ) with a significant level of ( 5% )</td>
</tr>
<tr>
<td>hazard computability</td>
<td>The level of effort required to perform PSHA</td>
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Various definitions for fractional order calculus have been pro-
posed; these approaches present different ways for deriving fractional
order operators and thereby their applications to a variable result in
different values. Caputo’s definition [45] is the most popular one for
everything applications because of its two main merits compared to
other definitions. First, Caputo’s definition stipulates that the frac-
tional derivative of a constant is zero, whereas other definitions (e.g. Grun-
wald–Letnikov and Riemann–Liouville) give unbounded values for
fractional derivatives of constants. Second, Caputo’s definition allows
physically interpretable initial conditions in dynamical systems (e.g.,
acceleration, velocity, displacement, etc.), whereas other definitions
require initial fractional responses.

Based on Caputo’s definition, the \( \alpha \)-derivative of a function \( f(t) \)
defined on the interval \([0, T]\) at time domain \( \alpha \in (0, T) \) is given by the
convolution integral expressed in Eq. (7), where \( \alpha \) is an integer number
satisfying \( p - 1 < \alpha \leq p \) and \( \Gamma(\cdot) \) is Euler’s Gamma function.

\[
\zeta D^\alpha f(t) = \begin{cases} 
\frac{1}{\Gamma(p-\alpha)} \int_0^t \frac{f(t)}{(t-s)^{\alpha-p+1}} \, ds, & \alpha > 0 \\
\frac{1}{\Gamma(p-\alpha)} \int_{t-p}^t \frac{f(t)}{(t-s)^{\alpha-p+1}} \, ds, & \alpha < 0 
\end{cases}
\]  

(7)

Fractional differential operators can also be implemented in the
Laplace domain. The Laplace transform of Caputo’s definition is ex-
pressed in Eq. (8) [45], where \( F(s) \) is the Laplace transform of \( f(t) \) and \( s \)
refers to the Laplace transform variable.

\[
\mathcal{L}^{-1}_\alpha D^\alpha f(t) = \left\{ \begin{array}{ll}
\mathcal{L}^{-1}_\alpha \left[ \sum_{k=0}^{p-1} \frac{\alpha_{k-1}}{\alpha_k} s^{\alpha_k} f^{(k)}(0) \right], & \alpha > 0 \\
\mathcal{L}^{-1}_\alpha \left[ \int_0^t (t-s)^{p-\alpha-1} f(s) \, ds \right], & \alpha < 0
\end{array} \right.
\]  

(8)

To implement the fractional operator, several approximation
algorithms have been proposed, among which, the Oustaloup’s
approximation [46] is widely used. In this approach, fractional derivatives
are approximated in the Laplace domain using a series of integer order
filters, which are defined to fit the frequency response of the fractional
operator within a defined frequency range of interest. To arrive at a
more precise estimate, a modification to the Oustaloup’s method pro-
posed by Xue et al. [47] is adopted in this study, which provides an
improved fit to the fractional operator transfer function at the bound-
aries of the frequency range of interest. Details of this modified ap-
proximation algorithm are presented in [31,47]. The adopted approxi-
mation algorithm of fractional order calculus in the Laplace domain
is implemented the SIMULINK environment in MATLAB [48].

3.2. Studied fractional order and traditional seismic intensity measures

Previous studies on seismic demand estimation of structures in li-
quefiable soils showed that velocity-based IMs and spectral acceler-
a at large periods perform well [25,49]. This paper extends such IMs
including \( PGV, HI, CAV, CAV_s \) and \( S_{a,20} \) to FO IMs. The following sub-
sections explain the definition of these FO IMs and their derivation
procedure.

3.2.1. Peak ground fractional response

One of the most common ways to quantify the intensity of a ground
motion is to identify the maximum ground response in terms of accel-
ceration (PGA), velocity \( (PGV) \) or displacement \( (PDG) \). However, con-
sidering the fractional order features exhibited in seismic wave propa-
gations [27], these intensities can be generalized to a fractional
response, using a real valued \( \alpha \) order integral of the recorded ground
acceleration. This FO IM, named peak ground fractional response (PGRα), was first introduced by Shafieezadeh et al. [31], as defined in Eq. (9).

\[ PGR_\alpha = \max_0^\infty (\int_0^\infty D_\alpha^2 \ddot{x}_\alpha(t) \, dt) \]  

(9)

where \( \ddot{x}_\alpha(t) \) is the recorded ground acceleration at time \( t \) and \( t_{\text{tot}} \) represents its duration. In this way, PGR_\alpha can be considered as a generalization to commonly used peak ground responses. Note that PGR_\alpha gives peak ground responses PGA, PGV and PGD for \( \alpha = 0 \), \( \alpha = 1 \), and \( \alpha = 2 \), respectively. In this study, a series of PGR_\alpha with real valued \( \alpha \in [-4, 1] \) that varies with a step of 0.1 are explored to assess the performance of nontraditional PGR_\alpha for PSDMs and identify the optimal value of \( \alpha \) as well.

Although rarely used in civil engineering, it is worth mentioning that PGR_\alpha when \( \alpha = 1 \), \( \alpha = -3 \) or \( \alpha = -4 \) represent the peak value of the so called jerk (derivative of acceleration) [50], absement (integral of displacement) [51] and abisty (double integral of displacement) [52], respectively. Specifically, jerk is a key measure for evaluating destructive effects of motions on a mechanism or the discomfort caused to passengers in a vehicle [53]; absement and abisty have been used in the design of a hydraulophone, which is an acoustic musical instrument played by direct physical contact with water flow [54].

3.2.2. Cumulative absolute fractional response

A novel FO IM proposed in this study is the cumulative absolute fractional response (CAR_\alpha), which is derived from CAV using the concept of fractional response. Eq. (10) presents the definition of this FO IM.

\[ \text{CAR}_\alpha = \int_0^{t_{\text{tot}}} \int_0^\infty D_\alpha^2 \ddot{x}_\alpha(t) \, dt \, dr \]  

(10)

where \( \int_0^\infty D_\alpha^2 \ddot{x}_\alpha(t) \, dt \) is the absolute value of the fractional integral of the recorded ground acceleration at time \( t \). CAR_\alpha increases with time such that it includes cumulative effects of ground motion, which are not captured by many amplitude-based IMs such as peak ground responses and spectral responses at specific periods. As to the seismic liquefaction that is associated with buildup of excess pore water pressures in soils during shaking, the cumulative effects may reflect the essential characteristics of liquefaction. Therefore, the proposed FO IM, CAR_\alpha may be an advanced index for seismic hazard prediction of liquefiable and laterally spreading ground as tested herein.

Kramer and Mitchell [21] proposed a modified CAV, represented by CAV_\alpha, that filters ground accelerations less than 5 cm/s² when their absolute values along the ground motion duration are integrated. Accordingly, a modified version of CAR_\alpha denoted by CAR_\alpha_\alpha was proposed in this study to explore the performance of cumulative absolute fractional order responses for ground accelerations larger than 5 cm/s². This FO IM is defined as follows

\[ \text{CAR}_\alpha = \int_0^{t_{\text{tot}}} (\chi) \int_0^\infty D_\alpha^2 \ddot{x}_\alpha(t) \, dt \, d\chi \]  

(11)

where \( \chi \) is equal to 0, CAR_\alpha and CAR_\alpha_\alpha provide the traditional CAV and CAV_\alpha, respectively. When \( \chi = 1 \), CAR_\alpha represents the traditional cumulative absolute displacement (CAD). In this study, a range of \(-4 \leq \alpha \leq 1\) for the value of \( \alpha \) is examined to explore the optimal one.

3.2.2.3. Fractional spectral acceleration

Another frequently used measure for intensity quantification of a ground motion is the spectral acceleration response (S_s) of a SDF system at periods of interest with linear damping features. In this study, S_s_\alpha_\alpha is generalized using fractional calculus in two distinct ways discussed as follows.

a) SDF system with fractional damping (S_s_\alpha_\alpha)

In the past few decades, the concept of fractional damping has been verified through analytical and experimental studies on a number of viscoelastic materials with fractional characteristics [33-35,39,40]. For dynamic problems in civil engineering, the equilibrium equation of motion of an SDF system with fractional damping can be expressed as

\[ m \ddot{x}(t) + c_\alpha D_\alpha^2 \ddot{x}(t) + kx(t) = p(t) \]  

where \( x(t) \) is the displacement response; \( p(t) \) is the forcing function; and \( m, c, k \) are mass, damping, and stiffness, respectively. The solution of Eq. (12) gives the pseudo spectral acceleration responses at periods of interest for fractionally damped SDF system, S_s_\alpha_\alpha_\alpha as expressed in Eq. (13).

\[ S_{s_\alpha_\alpha_\alpha} = \omega_\alpha^3 \max_0^\infty (|\chi(t)|) \]  

(13)

where \( \omega_\alpha \) is the period of interest and \( \omega_\alpha = 2\pi/T_\alpha \) is the corresponding circular frequency. In this study, a period of 2.0 s and a damping ratio of 5% is adopted to assess the performance of the proposed FO IM, S_s_\alpha_\alpha_\alpha. It is worth noting that S_s_\alpha_\alpha_\alpha gives traditional pseudo spectral acceleration of a conventional SDF system for \( \alpha = 0 \), where velocity-proportioned linear damping is utilized in Eq. (12). In this study, the performance of S_s_\alpha_\alpha_\alpha with \( \alpha \in [-2, 0] \) is examined. Note that the boundary values of \( \alpha = 0 \) and \( \alpha = 2 \) imply that the studied fractional damping varies from being displacement-proportional to acceleration-proportional. For \( \alpha \) values out of this range, there is lack of evidence to classify them to be ‘damping’ in a physical definition. Thus, this study investigates the range of \([-2, 0]\) for S_s_\alpha_\alpha_\alpha rather than \([-4, 1]\) that is for PGR_\alpha, CAR_\alpha and CAR_\alpha_\alpha as discussed above.

b) SDF system with fractional response (S_s_\alpha_\alpha_\alpha)

As mentioned before, structures with complex geotechnical characteristics that involve a wide range of nonlinear dynamic behaviors may not be best characterized by conventional dynamic features. In this regard, this paper assesses the seismic response of bridges in liquefiable soils and characterizes their structural response using fractional order models. To this end, in the second case, the fractional integral of the acceleration response of a conventional SDF system is used to identify the pseudo spectral acceleration at periods of interest (2.0 s in this study), as expressed in Eq. (14).

\[ S_{s_\alpha_\alpha_\alpha} = \omega_\alpha^3 \max_0^\infty (|\chi(t)|) \]  

(14)

where \( \chi(t) \) is the acceleration response of the well-known conventional SDF system subjected to the input ground motion. For the value equal to \( \alpha = 0 \), S_s_\alpha_\alpha_\alpha gives the common pseudo acceleration response of a conventional SDF system. In conjunction with the previously described S_s_\alpha_\alpha_\alpha it is worth noting that S_s_\alpha_\alpha_\alpha = S_s_\alpha_\alpha_\alpha - S_s_\alpha_\alpha. In this study, S_s_\alpha_\alpha_\alpha with \( \alpha \in [-4, 1] \) is explored to assess its performance in establishing the PSDMs.

3.2.4. Fractional spectrum intensity

As mentioned before, velocity related and spectrum-based traditional IMs such as H\i are found to be optimal IMs for probabilistic demand modeling of pile foundations in liquefiable soils [22]. One of the reasonable explanations is that H\i integrates the pseudo spectral velocity of a ground motion along a range of periods from 0.1 s to 2.5 s, which contains common frequency properties of soil-structure systems. In this regard, another unique FO IM is proposed in this study to generalize the conventional spectrum intensity with fractional characteristics. Similar to the above described two types of fractional spectral accelerations, two cases of fractional spectrum intensities are proposed and explained as follows.

a) Fractionally damped SDF system (S_d_\alpha_\alpha)

In the first case, pseudo spectral velocity of a fractionally damped SDF system derived from Eq. (12) is utilized to establish the novel
FO IM, $SI_{40}$ given in Eq. (15).

$$SI_{40} = \int_{0}^{2.5} S_{40} dT = \int_{0}^{2.5} (2\pi/T)^{-(\alpha-1)} \max(|\xi|) dT$$  \hspace{1cm} (15)

where $S_{40}$ is the pseudo spectral velocity derived from the displacement response, $x(t)$ of the fractionally damped SDF system. When $\alpha$ value equals $-1$, $SI_{40}$ equals $HI$. Similar to $SI_{40}$, a range of $[-2, 0]$ for the $\alpha$ values of $SI_{40}$ are explored to assess the performance of FO IMs in PSDMs.

b) SDF system with fractional response ($SI_{\alpha}$)

In the second case, fractional pseudo spectral velocity of a conventional SDF system is obtained to form another novel FO IM, $SI_{\alpha}$ expressed in Eq. (16).

$$SI_{\alpha} = \int_{0}^{2.5} S_{\alpha} dT = \int_{0}^{2.5} (2\pi/T)^{-(\alpha-1)} \max(|\xi|) dT$$  \hspace{1cm} (16)

where $S_{\alpha}$ is the fractional pseudo spectral velocity derived from the acceleration response, $\dot{x}(t)$ of the linear SDF system. In this case, $SI_{\alpha}$ with an $\alpha$ value of $-2$ corresponds to the traditional IM, $HI$. Also in this study, a range of $[-4, 1]$ for a values of $SI_{\alpha}$ are explored to assess their performances.

For completeness, PSDM studies on traditional IMs as well as FO IMs are performed for comparisons. Table 2 summarizes the studied IMs. Note that a values in the table are fractional orders of integrals with respect to the input ground acceleration or acceleration responses of SDF systems.

4. Description of numerical models

4.1. Overview of adopted finite element models

A coupled soil-bridge model considering geometric parameters and structural and geotechnical material uncertainties is adopted in this study to identify the optimal fractional order IMs for probabilistic seismic demand modeling of bridges in laterally spreading ground. As illustrated in Fig. 1(a), a two-dimensional reinforced concrete (RC) extended pile-shaft is linked to a two-dimensional multilayered soil column with a gently sloping degree (nonliqueifiable crust overlying liquefiable loose and dense sands). Common elastomeric rubber bearings (ERBs) are used to connect the pile-shaft top with the superstructure as represented by a lumped mass. This simplified model represents typical multispans highway bridges or viaducts with multiple individual bents that have a fairly uniform distribution of strength and stiffness among them in the transverse direction. For this reason, abutments are not considered in this study. More specifically, each bent is supported by a single large-diameter extended pile-shaft embedded into multilayered and gently sloping liquefiable grounds that are susceptible to lateral spreading under earthquakes. It is worth noting that this simplified modeling process has been adopted in previous studies [55,56].

One of the critical issues for laterally spreading ground is the prediction of shear localization-induced displacement-discontinuity between the loose sand and overburden clay layers, and its impact on responses of piles and other structural components. To this end, Wang et al. [57] recently proposed a simplified modeling technique based on the soil material library in OpenSees [41]; that is a soft interlayer built underneath the clay crust, which is characterized by a specific thickness ($h_{\alpha}$) representing the depth of shear localization and a corresponding low reference shear modulus ($G_{\alpha,soft}$). According to the study, $h_{\alpha}$ was found to fall in a range of 0.1–1.0 m based on physical observations from several centrifuge tests [58–60]. $G_{\alpha,soft}$ depends on $h_{\alpha}$ with a relationship expressed as below.

$$G_{\alpha,soft} = 40 \cdot e^{3-h_{\alpha}}$$  \hspace{1cm} (17)

where $e$ is the base of the natural logarithm. This modeling technique has been validated through simulations of three centrifuge tests, in which different types of pile foundations including extended pile-shaft, two-pile group and six-pile group foundations were embedded into gently sloping grounds that are similar to the soil conditions in the present study. Details on the centrifuge tests and associated validation results can be found in [57]. Due to the lack of information about the uncertainty in $h_{\alpha}$, deterministic values of $h_{\alpha} = 0.5$ m and $G_{\alpha,soft} = 85$ kPa are taken in this study for simplicity.

4.2. Geometric parameters of the bridge and material uncertainties

To assess the impact of geometric variables on FO optimal IMs, ten scenarios (abbreviated as S0 to S9 hereinafter) are considered as summarized in Fig. 1(b). The base scenario (S0) represents a 2 m-diameter (D) extended pile-shaft supported highway bridge with column height ($L_c$) of 8 m and embedded into a 4° sloping ground ($\theta$) of 5 m-thick clay deposit ($H_c$) overlying 4.5 m-thick loose sand layer ($H_l$) and overlying 20 m-thick dense sand layer. An axial compressive ratio ($R_{ax}$) of 15% is adopted to represent common cases in practice [61]. The superstructure mass is then determined by $M_{ss} = R_{ax} A \cdot \varepsilon_{cu,cover} / \varepsilon_{c,cover}$, where $\varepsilon_{cover}$ is compressive strength of unconfined concrete and $A$ is the gross section area. Scenario S1-S8 include variations of pile-column diameter, column height, soil layer profile and ground sloping degree. S9 represents a comparative scenario of acilnic ground ($\theta = 0°$) such that no lateral spreading effect is involved.

Soil-pile-structure systems normally have high levels of uncertainties in their behavior. Therefore, this study considers material uncertainties of soils (undrained shear strength of clay, $S_u$, and relative density of loose sand, $D_r$) and piles ($f_{cover}$, peak strength corresponding strain, $\varepsilon_{c,cover}$ and ultimate strain, $\varepsilon_{c,cover}$) of the concrete cover and yielding stress, $f_p$, modulus of elasticity, $E$, and hardening ratio, $b$ of the longitudinal rebar). The probability distribution type, mean value, coefficient of variation (COV), and the corresponding reference of each considered random variable are listed in Table 3. Note that each pair of these variables is regarded as independent except for the pair of $f_{cover}$ and $\varepsilon_{c,cover}$ where a correlation coefficient of 0.8 is adopted based on engineering judgment [62] to ensure $\varepsilon_{c,cover}$ remains larger than $\varepsilon_{c,cover}$. 170 cases for each scenario are generated using a Latin hypercube sampling (LHS) technique [63], which are randomly paired with the

<table>
<thead>
<tr>
<th>IM</th>
<th>Definition</th>
<th>Studied a value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PGA</td>
<td>Peak ground acceleration</td>
<td>0</td>
</tr>
<tr>
<td>PGV</td>
<td>Peak ground displacement</td>
<td>1</td>
</tr>
<tr>
<td>CAV</td>
<td>Cumulative absolute velocity</td>
<td>2</td>
</tr>
<tr>
<td>CAD</td>
<td>Cumulative absolute displacement</td>
<td>1</td>
</tr>
<tr>
<td>CAS</td>
<td>Cumulative absolute velocity with acceleration beyond 5 cm/s²</td>
<td>0</td>
</tr>
<tr>
<td>$S_{40}$</td>
<td>Spectral acceleration at 2.0 s</td>
<td>$-1°$ or $-2°$</td>
</tr>
<tr>
<td>$HI$</td>
<td>Hounser intensity, also known as the response spectrum intensity</td>
<td>$-1°$ or $-2°$</td>
</tr>
</tbody>
</table>

For FO IMs:
- $PG_{\alpha}$: Peak ground fractional response
- $CAR_{\alpha}$: Cumulative absolute fractional response
- $CAR_{\alpha}$: Cumulative absolute fractional response with acceleration beyond 5 cm/s²
- $S_{40-\alpha}$: Spectral acceleration at 2.0 s for fractionally damped SDF system
- $S_{40-\alpha}$: Spectral acceleration at 2.0 s for linear SDF system with fractional response
- $SI_{\alpha}$: Spectrum intensity for fractionally damped SDF system
- $SI_{\alpha}$: Spectrum intensity for linear SDF system with fractional response

* Corresponds to $S_{40-\alpha}$ and $SI_{\alpha}$.
* Corresponds to $S_{40-\alpha}$ and $SI_{\alpha}$.
selected 170 ground motions (described later). The reported upper and lower boundaries of the random variables in Table 3 are the maximum and minimum values of random realizations of variables generated through the LHS technique.

### 4.3. Modeling of soil profiles and extended pile shafts

Two soil constitutive models including PIMY and PDMY material models [68] are adopted for modeling clay and sand, respectively. The soil models are assigned to four-node QuadUP elements that can simulate the response of solid-fluid coupled materials under cyclic excitation [69]. Constitutive parameters of the 0.5 m meshed soil column are determined based on the modeling approach in [57]. The nodes on opposite sides of the soil elements (at the same depth) are tied together to simulate a pure-shear condition. Horizontal static forces associated with the ground slope are assigned to the clay elements to represent the lateral spreading effect. P-y and t-z springs are spaced at 0.5 m consistent with the mesh of the soil column. The pile-tip is linked to the soil column using a q-z spring. More details on constitutive models of the p-y, t-z and q-z springs and associated approaches for parameter determinations can be found in [57].

The RC extended pile-shaft is modeled using displacement-based beam-column element with fiber section discretized into 0.5 m in depth. The deck is modeled by a 0.5 m-length rigid elastic beam-column element with a lumped mass. The Corotational Transformation command in OpenSees [41] is used to account for the P-Δ effect. The fiber section is meshed into 1 and 8 segments in the radial direction for concrete cover and core, respectively, and 10 segments in the circular direction (Fig. 1(e)). The mesh is found to generate almost identical curvature responses compared with a very fine but numerically time-consuming discretization (3 and 24 segments in the radial direction for cover and core, respectively, and 18 segments in the circle direction). The width of the concrete cover is set as 0.1 m. A longitudinal reinforcement ratio of 2% and a stirrup reinforcement ratio of 1% are adopted to determine the constitutive parameters of concrete core based on Mander et al. [70]. The concrete fibers are then assigned using the Concrete04 material [70] (Fig. 1(d)). The initial modulus of elasticity, $E_c$ (MPa) for normal weight concrete is determined as $E_c = 4.700f_c^c$ [71], where $f_c$ (MPa) is the compressive strength of the concrete cover or core. The longitudinal rebars are modeled by the Steel02 material [72] displayed in Fig. 1(e).

The characteristic strength $f_{by}$ can be obtained by assuming (1) a ‘yielding’ deformation $\delta_y = 0.075t_e$ and (2) a pre-‘yielding’ stiffness $k_1 = 10k_2$ [73]. In this study, $G_0 = 1200$ kN/m and $t_e = 76$ mm is adopted based on engineering judgment, whereas $A_0$ is determined based on [74] associated with the superstructure weight; that is $A_0 = M_s(\alpha_s/\alpha_f)$, where $\alpha_f = 11$ MPa is the prescriptive vertical bearing capacity and $\alpha_f = 1.5$ is the assumed safety factor. Since this study concentrates on the transverse response, this constitutive model is assigned to the transverse degree-of-freedom of the zero-length element that represents the ERBs (see Fig. 1), whereas the rotational degree-of-

---

**Table 3**

Properties of considered soil and pile material uncertainties.

<table>
<thead>
<tr>
<th>Variable (Unit)</th>
<th>Distribution</th>
<th>Mean</th>
<th>COV</th>
<th>Reference</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_s$ (kPa)</td>
<td>Lognormal</td>
<td>40</td>
<td>32%</td>
<td>[64]</td>
<td>24</td>
<td>126</td>
</tr>
<tr>
<td>$D_s$ (%)</td>
<td>Normal</td>
<td>40</td>
<td>19%</td>
<td>[64]</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>$f_{c,cover}$ (MPa)</td>
<td>Lognormal</td>
<td>34.00</td>
<td>18%</td>
<td>[65]</td>
<td>25.14</td>
<td>64.90</td>
</tr>
<tr>
<td>$e_{c,cover}$</td>
<td>Lognormal</td>
<td>0.002</td>
<td>20%</td>
<td>[62]</td>
<td>0.0014</td>
<td>0.0049</td>
</tr>
<tr>
<td>$e_{cu,cover}$</td>
<td>Lognormal</td>
<td>0.005</td>
<td>20%</td>
<td>[66]</td>
<td>0.0036</td>
<td>0.0117</td>
</tr>
<tr>
<td>$f_y$ (MPa)</td>
<td>Lognormal</td>
<td>400</td>
<td>5%</td>
<td>[67]</td>
<td>357</td>
<td>479</td>
</tr>
<tr>
<td>$E_s$ (GPa)</td>
<td>Lognormal</td>
<td>200</td>
<td>3.3%</td>
<td>[62]</td>
<td>188</td>
<td>232</td>
</tr>
<tr>
<td>$b$</td>
<td>Lognormal</td>
<td>0.01</td>
<td>20%</td>
<td>[62]</td>
<td>0.0072</td>
<td>0.0208</td>
</tr>
</tbody>
</table>

---

**Fig. 1.** Adopted bridge models: (a) schematic illustration, (b) studied scenarios, (c) section mesh, (d) concrete constitutive model, (e) steel constitutive model and (f) ERB constitutive mode.
freedom is fixed since the deck and column are expected to rotate synchronously in the transverse direction.

4.4. Boundary condition and solution scheme

The clay surface nodes are allowed to drain, whereas other soil nodes are set as impervious. Soil bottom nodes are fixed in the horizontal and vertical directions to impose uniform excitations (Fig. 1). In the time history analysis, Krylov-Newton algorithm [75] and β-Newmark integrator [76] with parameters β = 0.3025 and γ = 0.6 are used to solve the matrix equations of the finite element models. The convergence tolerance on the norm of the displacement residuals is set to \(10^{-4}\). Based on previous studies on soil shear stress-strain hysteretic curves [77], a relatively low level of stiffness-proportional damping coefficient of 0.006 is employed to enhance the numerical stability [57,78].

4.5. Selected ground motions

The embedment depth of the extended pile-shaft is 30 m. A set of 80 unscaled horizontal ground motions for rock sites of California selected by Baker and Shahi [79] is adopted in this study. The acceleration of these records is doubled to obtain another 80 ground motions. In addition, another set of 10 scaled strong real site ground motions (PGA around 1.0 g) for the site of Los Angeles provided by Somerville et al. [80] is added. This larger set of ground motions helps to realize various seismic behaviors of the bridge models, from the elastic response to failure. Fig. 2(a) shows acceleration response spectra of the adopted 170 records together with their median values. The first-mode period of the studied bridge models ranges from 1.23 s to 3.75 s. The relatively large values in this range are partly because of the considered soil material uncertainties (i.e. \(S_s\) and \(D_s\)) that produce cases with quite soft clay and/or loose sand. Additional modal analyses on the bridge models with fixed base at the soil surface (neglecting SFSI) exhibit a first-mode period range of 0.52 s ~ 1.90 s. Fig. 2(b) illustrates statistics of the magnitude and source distance properties of the selected ground motions.

5. PSDM studies and optimal IM results

This section presents and discusses results of PSDM studies for the IMs mentioned in Section 3.2. Considered engineering demand parameters (EDPs) are discussed first, followed by the results for the base scenario S0 for the optimal traditional IMs as well as the proposed FO IMs. Then, impacts of the geometric parameters of the bridge models on the optimal \(\alpha\) values of the studied FO IMs are investigated.

5.1. Studied engineering demand parameters

EDPs are response measures used for assessing damage to bridge components under earthquakes. For the bridge models in this study (Fig. 1), critical EDPs include maximum bearing deformation (\(\delta_b\)), maximum column drift ratio (\(\gamma_{c,max}\)), maximum pile curvature (\(\psi_p\)), maximum soil surface displacement (\(\Delta_s\)), and residual column drift ratio (\(\gamma_{c,res}\)). Note that the column drift ratio is calculated by dividing the difference of lateral displacements between the column top and the column at soil surface by the length of the column. \(\delta_b\), \(\gamma_{c,max}\) and \(\psi_p\) directly reflect damage suffered during earthquakes. \(\gamma_{c,res}\) is an empirical index for post-earthquake repairability estimations [81]. \(\Delta_s\) is a measure of damage induced by liquefied and laterally spreading ground, which approximately reflects the kinematic loads acting on the pile. Note that the maximum soil surface displacement is practically identical to its residual counterpart in laterally spreading ground. A preliminary study on the correlations between these EDPs shows that \(\psi_p\) is highly correlated with \(\gamma_{c,max}\) (i.e. coefficient of determination, \(R^2 = 0.96\) for scenario S0) while other EDPs are not well correlated with each other [25]. This is because \(\gamma_{c,max}\) and \(\psi_p\) are both influenced by kinematic loads as well as inertia forces, whereas \(\delta_b\) is mainly triggered by inertia forces and \(\Delta_s\) reflects kinematic loads only. It is reasonable to conclude that using \(\gamma_{c,max}\) and \(\psi_p\) as EDPs will yield very similar results in assessing the performance of IMs in the PSDMs of the studied models. Therefore, PSDM studies in terms of \(\psi_p\) are not presented for conciseness. Table 4 summarizes the studied EDPs in this research.

5.2. Results of base scenario S0

5.2.1. PSDM studies of traditional IMs

Table 5 presents values of various measures used to assess the quality of intensity measures for the considered set of traditional IMs in scenario S0. It is clear that PGV is the prime IM for the peak structural demand parameters (i.e. \(\gamma_{c,max}\) and \(\delta_b\)). This finding is consistent with the conclusions of previous studies on a three-span continuous steel girders bridge in aclinie liquefiable soil [23]. HI and \(S_s\) are the next most preferred IMs for both \(\gamma_{c,max}\) and \(\delta_b\). As for the residual structural demand parameter, \(\gamma_{c,res}\) and the soil demand parameter, \(\Delta_s\), HI tends to be the best choice, followed by \(S_s\) and CAV. In contrast, PGD underperforms for all the EDPs, which is consistent with the results of past studies on portfolios of highway bridges founded on grounds without liquefactions [8,31]. In addition, PGD and CAF are found to be insufficient for EDPs of \(\gamma_{c,res}\) and \(\Delta_s\). Hereafter, PGV for \(\gamma_{c,max}\) and \(\delta_b\) and HI for \(\gamma_{c,res}\) and \(\Delta_s\) are set as the base IMs for quantitative comparison to the FO IMs, because they are found to be the best choices among the commonly used IMs.

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![Fig. 2.](image-url) Properties of selected ground motions: (a) acceleration response spectra (5% damping ratio) and (b) statistics of magnitude and source distance.
Table 4

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>EDP</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>δb</td>
<td>Maximum bearing deformation</td>
<td>m</td>
</tr>
<tr>
<td>γ_{c,max}</td>
<td>Maximum column drift ratio</td>
<td></td>
</tr>
<tr>
<td>φp</td>
<td>Maximum pile curvature</td>
<td>1/m</td>
</tr>
<tr>
<td>δp</td>
<td>Maximum soil surface displacement</td>
<td>m</td>
</tr>
<tr>
<td>γ_{c}</td>
<td>Residual column drift ratio</td>
<td></td>
</tr>
</tbody>
</table>

5.2.2. PSDM studies of FO IMs

Fig. 3 shows the plots of the six metrics with respect to the α ranges ([-4, 1] or [-2, 0]) for γ_{c,max} for scenario S0. From a quick view of Fig. 3, it is seen that curves of PGRα, CARα and CAR_{5α} derived from acceleration time histories follow the same trends, whereas S_{ad-20α}, S_{ad} and S_{SR} derived based on spectral responses exhibit other types of trends. It is worth noting that the results in Fig. 3 at integer α values such as −2, −1 and 0 yield identical results as those for corresponding traditional IMs previously listed in Table 5. In particular, S_{ad-20-1}, S_{ad-20-2} and S_{ad} must yield the same results across all these characteristic measures. Similarly, S_{ad-20-1}, S_{ad-20-2} and HI should result in identical values of these characteristic measures. Fig. 4 displays this concept using the plot of efficiency index, β_{p|IM} in terms of α values for the maximum column drift ratio, γ_{c,max}.

Again seen from Fig. 3(a) to (d), optimal α values observed on the basis of β_{p|IM} are identical to the R^2-based results, as reasonably expected per their statistical definitions. As for comparisons among characteristic measures of b, β_{p|IM} and γ, the optimal α values for most of the studied FO IMs appear to be very close. For instance, PGR_{α} exhibits the maximum b at α = −1.2, while it attains the minimum β_{p|IM} at α = −1.4, leading to a minimum γ = β_{p|IM} / b at α = −1.3. The same trends occur in cases of CAR_{α}, CAR_{5α}, S_{ad-20α} and S_{SR}. However, exceptions are observed in cases of S_{ad-20} and S_{SR} in which large differences of optimal α values exist between b and β_{p|IM}. Specifically for S_{ad-20α}, maximum b corresponds to α = −1.3 while minimum β_{p|IM} takes place at α = −3.4. Nevertheless, due to the minor variation of b with respect to the α values in the range of [-4, -1], minimum γ stays at α = −3.4. As to S_{SR}, a reversed trend is detected where the optimal α values are located at the boundary of the studied α range of [-2, 0]. For example, the prime α value for b is 2 while it becomes 0 for β_{p|IM}. As a result, quite close values of γ are obtained at α = −2 and 0 in Fig. 3(c). In terms of γ that considers both the practicability and efficiency properties, S_{ad-3.3} is a superior IM for γ_{c,max}, followed by S_{ad-0.0}, CAR_{0.7} and PGR_{1.3}. Note that the optimal α = 0 for CAR_{5α} corresponds to the traditional IM, CAV_{5α}, indicating that the proposed FO IM, CAR_{5α} does not perform better than the traditional one. Thus, quantitative comparisons between CAR_{5α} and traditional IMs are not presented hereinafter.

According to the sufficiency results shown in Fig. 3(e) and (f), all the optimal α values based on b, β_{p|IM}, γ, and R^2 are found to be sufficient (i.e. both M_p-value and R_{p-value} exceed 0.05). Specifically, the previously detected optimal α values for PGR_{α}, S_{ad-20α} and S_{SR} correspond to large values of M_p-value and R_{p-value} which further indicates the viability of these IMs.

For easy inspections, Fig. 3 also marks the previously obtained optimal traditional IM (i.e., PGV, also known as PGR_{α} at α = −1, for γ_{c,max}). It is seen that the optimal FO IMs (e.g., PGR_{1.3}, CAR_{0.7} and S_{ad}) outperform the traditional one. More specifically, Table 6 presents comparisons of b, β_{p|IM}, γ, R^2, M_p-value and R_{p-value} for the six FO IMs at their-based optimal α values as well as the abovementioned traditional IMs for all of the studied EDPs. From a quick inspection, it can be seen that the proposed S_{ad} at α = −3.3 and −3.2 are optimal FO IMs in terms of γ_{c,max} and δp, respectively, whereas S_{ad-20α} at α = −1.9 and −2.0 are optimal FO IMs in terms of γ_{c} and Δp, respectively. All of these four IMs outperform the optimal traditional ones. In addition, PGR_{1.3} and S_{ad-0.0} also perform quite well in terms of γ_{c,max}. It is worth noting that γ_{c,max} and δp are peak structural demands, while γ_{c} is the residual structural demand and Δp is the peak ground demand which is almost identical to the residual ground demand for liquefied and laterally spreading soils. For most cases, optimal α values for a FO IM tend to be quite close or even identical for peak and residual demands. For example, optimal α value for PGR_{α} in terms of γ_{c,max} is α = −1.3, while the optimal α value for δp is α = −1.2. Nevertheless, it should be noted that minor variations in α values for a particular EDP (e.g. α = −1.3 or −1.2 for γ_{c,max}) produce very close results, as illustrated in Fig. 3.

In this regard, it is reasonable to select an identical optimal α value for different EDPs, for convenience.

For the base scenario S0, S_{ad-3.3} tends to be an optimal IM in terms of γ_{c,max} and δp, whereas S_{ad-1.9} is a prime IM for γ_{c} and Δp. Moreover, γ_{c,max} and δp share the same optimal value of α = −0.2 for S_{ad-0.0}. Similarly, optimal CAR_{α} occurs at α = −0.4 for both γ_{c} and Δp. However, an exception is detected for S_{ad-20α} in cases of γ_{c} and Δp, where optimal α = −0.3 and −1.0, respectively. This result implies that optimal α values for structural and soil demands may be quite different for a particular FO IM such as S_{ad-20α}. In addition, most of the studied FO IMs pass the p-value-based sufficiency check, except for

Table 5

| IM       | b     | β_{p|IM} | γ | R^2 | M_p-value | R_{p-value} | IM       | b     | β_{p|IM} | γ | R^2 | M_p-value | R_{p-value} |
|----------|-------|----------|---|-----|----------|-------------|----------|-------|----------|---|-----|----------|-------------|
| PGA      | 0.62  | 0.66     | 1.06 | 0.32 | 0.08     | 0.82        | PGA      | 0.90  | 1.28     | 1.42 | 0.21 | 1.00     | 0.69        |
| PGV      | 0.88  | 0.47     | 0.53 | 0.66 | 0.18     | 0.90        | PGV      | 1.08  | 1.20     | 1.11 | 0.30 | 0.85     | 0.62        |
| PGD      | 0.65  | 0.51     | 0.79 | 0.58 | 0.07     | 0.06        | PGD      | 0.68  | 1.29     | 1.90 | 0.20 | 0.33     | 0.28        |
| CAV      | 0.86  | 0.60     | 0.70 | 0.43 | 0.95     | 0.92        | CAV      | 1.15  | 1.25     | 1.09 | 0.24 | 0.47     | 0.66        |
| CAV_{5}  | 0.81  | 0.61     | 0.75 | 0.44 | 1.00     | 0.90        | CAV_{5}  | 1.09  | 1.25     | 1.14 | 0.24 | 0.48     | 0.67        |
| CAD      | 0.86  | 0.46     | 0.54 | 0.67 | 0.08     | 0.27        | CAD      | 0.96  | 1.23     | 1.28 | 0.26 | 0.29     | 0.39        |
| S_{ad}   | 0.77  | 0.46     | 0.59 | 0.67 | 0.10     | 0.67        | S_{ad}   | 0.91  | 1.21     | 1.33 | 0.29 | 0.88     | 0.66        |
| HI       | 0.88  | 0.46     | 0.53 | 0.66 | 0.05     | 0.73        | HI       | 1.14  | 1.16     | 1.01 | 0.35 | 0.99     | 0.69        |

Note: Bold faced values indicate more practical, efficient, proficient, or sufficient IMs. Italic values indicate IMs do not pass the sufficiency check with a significant level of 5%.
CAR that is insufficient in cases of $\delta_b$ and $\Delta_s$ as demonstrated in Table 6. Furthermore, $p$-values for a majority of the FO IMs are found to be close to or larger than that for the optimal traditional IMs. Therefore, it is reasonable to conclude that the proposed FO IMs maintain the sufficiency properties compared with the optimal traditional IMs.

To elaborate the superiority of the studied FO IMs for probabilistic seismic demand modeling of bridges in liquefiable and laterally spreading ground, Table 7 presents percentage changes of the characteristic measures of an optimal IM (in terms of various EDPs) with respect to their optimal traditional IMs. Positive values indicate percentage increases, whereas negative values indicate percentage decreases. The goal of pursuing a superior FO IM compared with the optimal traditional one is to increase $b$ and $R^2$ and reduce $\beta_{D|IM}$ and $\zeta$. It can be clearly seen from Table 7 that, in general, most of the studied FO IMs (with their respective optimal $\alpha$ values) outperform their optimal traditional IMs. In the case of $\gamma_{c,max}$, $S_{nd,3.3}$ is the optimal IM, followed by $S_{il,0.0}$, CAR, and $PGR$. Specifically, $\beta_{D|IM}$ and $\zeta$ are reduced by as much as 17% and 21%, respectively, relative to PGV; meanwhile $b$ and $R^2$ are increased by as much as 6% and 15%, respectively. Other FO IMs for $\gamma_{c,max}$ yield similar (but relatively lower) improvements except for the fractional spectral accelerations ($S_{ad,20.2}$ and $S_{ar,20.3}$) where $b$ is reduced by as much as 10% compared to PGV. Moreover, $S_{ad,20}$ and $S_{ar,20}$ (with their respective optimal $\alpha$ values) fail to increase $b$ for all other EDPs. These IMs also appear less appropriate than $HI$ for $\gamma_{cr}$ and $\Delta_s$. However for these two EDPs ($\gamma_{cr}$ and $\Delta_s$), the proposed $S_{da}$ with optimal $\alpha$ values of $-1.9$ and $-2.0$, respectively, show noticeable

Fig. 3. Plot of (a) slope, (b) dispersion, (c) modified dispersion, (d) coefficient of determination, (e) $p$-value with respect to magnitude and (f) $p$-value with respect to source distance versus the fractional order for maximum column drift ratio from scenario S0 as the demand measure.
- There may be two reasons as to why the FO IMs perform better for $\gamma_{\text{res}}$ and $\Delta_\gamma$ compared to the cases of $\gamma_{\text{max}}$ and $\delta_\beta$. First, the proposed $SI_{\text{res}}$ is found to be an optimal IM for $\gamma_{\text{res}}$ and $\Delta_\gamma$, followed by $SI_{0.0}$ and $PGR_{1.3}$, whereas $SI_{0.1.9}$ appears to be the best choice for $\gamma_{\text{res}}$ and $\Delta_\gamma$, followed by $SI_{-2.7}$ and $\text{CAR}_{-0.4}$. The seismic demands of these models may not all be highly correlated with $\gamma_{\text{res}}$ and $\Delta_\gamma$, whereas $\Delta_\gamma$ with different option in most scenarios. This may be somewhat attributed to the fact that the analyzed soil-bridge models considering geometric parameters and material uncertainties have a relatively wide range of fundamental periods (as illustrated in Fig. 2(a)). The seismic demands of these models may not all be highly impacted by the spectral value at the same discrete period (e.g. 2.0 s). Also, the studied soil-bridge systems in the presence of liquefaction and lateral spreading respond in a highly non-linear fashion under seismic loading. Thus, the initial periods of the soil-bridge systems may not dictate the response because of elongation of natural periods due to degradation and softening in structures and soils in the presence of liquefaction. In addition, $\text{CAR}_\alpha$ is not a superior IM for the case of $\delta_\beta$. $PGR_\alpha$ underperforms compared to $HI$ and almost all other FO IMs for the case of $\Delta_\gamma$.

5.3. Impact of geometric parameters

5.3.1. Proficiency comparisons among different scenarios

To further demonstrate the effectiveness of the studied FO IMs, Fig. 5 plots the percentage changes of modified dispersions, $\zeta$, with respect to their corresponding values for optimal traditional IMs across all ten scenarios. Note that, $\gamma_{\text{res}}$-indexed proficiency is a composite measure of efficiency and practicality. Negative values indicate a decrease in the modified dispersion and therefore the superior performance of FO IMs, while positive values indicate otherwise. It can be clearly seen that $SI_{\text{res}}$ (with their optimal $\alpha$ values presented later) tends to be the best choice for cases of $\gamma_{\text{max}}$ and $\delta_\beta$ (providing an average improvement by as much as 20% and 8%, respectively), followed by $SI_{\alpha}$, $PGR_\alpha$ and $\text{CAR}_\alpha$; whereas in the cases of $\gamma_{\text{res}}$ and $\Delta_\gamma$, $SI_{\alpha}$ is the preferred option in most scenarios (improving $\zeta$ as much as 10–20%), followed by $\text{CAR}_\alpha$ and $SI_{\alpha}$. By comparison, the fractional spectral accelerations ($S_{\alpha}$ and $S_{\alpha.20a}$) fail to decrease $\zeta$ in most scenarios. This may be somewhat attributed to the fact that the analyzed soil-bridge models considering geometric parameters and material uncertainties have a relatively wide range of fundamental periods (as illustrated in Fig. 2(a)). The seismic demands of these models may not all be highly impacted by the spectral value at the same discrete period (e.g. 2.0 s). Also, the studied soil-bridge systems in the presence of liquefaction and lateral spreading respond in a highly non-linear fashion under seismic loading. Thus, the initial periods of the soil-bridge systems may not dictate the response because of elongation of natural periods due to degradation and softening in structures and soils in the presence of liquefaction. In addition, $\text{CAR}_\alpha$ is not a superior IM for the case of $\delta_\beta$. $PGR_\alpha$ underperforms compared to $HI$ and almost all other FO IMs for the case of $\Delta_\gamma$.

In order to verify the rigor of this method, the stability of optimal $\alpha$ values across all ten scenarios are assessed. Fig. 6 plots variations of the optimal $\alpha$ values for the superior FO IMs for different EDPs. From a quick inspection of Fig. 6, the variation of optimal $\alpha$ values for $PGR_\alpha$, $\text{CAR}_\alpha$ and $SI_{\alpha}$ is small, while the variation for $SI_{\alpha}$ is large and the optimal $\alpha$ values vary between $-2$ and $0$. For example, optimal $\alpha$ values of $PGR_\alpha$ for $\gamma_{\text{max}}$ practically remain constant at $\alpha = -1.3$ except for scenarios S3 and S4, in which optimal values of $\alpha$ are $-1.1$ and $-1.6$, respectively. Recall the geometric properties of scenario S3 and S4 where the aboveground height is 4 m and 12 m, respectively. Also, recall the concept that for structures with short periods, seismic responses are highly correlated with $PGA$, whereas responses of structures with long periods are more likely correlated with $PGD$ [82]. Therefore, it is reasonable that scenario S4 with a larger aboveground height that extends the predominant period of the bridge tends to have an optimal $\alpha$ close to $PGD$ ($\alpha = -2$ for $PGR_{\alpha}$), while scenario S3 with a shorter
optimal traditional IMs, respectively. FO IM for demand models of the smallest values for pro
next appropriate IMs for these EDPs. In addition, parameters.
generally approaching Fig. 6 for the same EDPs indicate that the optimal value of aboveground height should have an optimal PGR, with the value generally approaching PGA. The same trends are observed in PGR for γc,res and δb, in CAR for γc,max, and in Sld for structural demand parameters.

From results presented in Fig. 5, it can be concluded that Sld yields the smallest values for proficiency for γc,res and δb. Moreover, results in Fig. 6 for the same EDPs indicate that the optimal value of a for Sld remains quite stable. Therefore, it is reasonable to conclude that Sld,0.9 is an optimal IM for γc,res and Δb measures. CAR,0.4 and Sld,2.7 are the next appropriate IMs for these EDPs. In addition, Sld,3.3 is the optimal FO IM for demand models of γc,max and δb, followed by Sld,2.0 and PGR,1.3. Sample PSDMs using FO IMs are shown in Fig. 7, including Sld for γc,max in scenario S0 and Sld for Δb in scenario S5.

Table 7
Percentage change in metric measures of optimal FO IMs with respect to optimal traditional IMs in terms of various EDPs for scenario S0.

<table>
<thead>
<tr>
<th>IM</th>
<th>b</th>
<th>βD</th>
<th>IM</th>
<th>γ</th>
<th>R²</th>
<th>IM</th>
<th>b</th>
<th>βD</th>
<th>IM</th>
<th>γ</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGR,1.3</td>
<td>γc,max (relative to PGV)</td>
<td>0</td>
<td>−10</td>
<td>−9</td>
<td>9</td>
<td>PGR,0.6</td>
<td>6</td>
<td>1</td>
<td>−4</td>
<td>−4</td>
<td></td>
</tr>
<tr>
<td>CAR,0.7</td>
<td>−10</td>
<td>−5</td>
<td>−12</td>
<td>3</td>
<td>CAR,0.4</td>
<td>9</td>
<td>4</td>
<td>−4</td>
<td>−15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sld,0</td>
<td>9</td>
<td>−14</td>
<td>−4</td>
<td>12</td>
<td>2</td>
<td>Sld,0.9</td>
<td>10</td>
<td>−1</td>
<td>−9</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Sld,0.6</td>
<td>5</td>
<td>−11</td>
<td>−15</td>
<td>10</td>
<td>15</td>
<td>Sld,2.7</td>
<td>3</td>
<td>−1</td>
<td>−4</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Note: All values in the table are percentage. Bold faced values mean attaining the goal to increase b, R² and reduce βD and γ by using FO IMs compared with optimal traditional IMs, respectively.

5.3.2. Sufficiency comparisons among different scenarios
To verify the sufficiency properties of the abovementioned optimal FO IMs, Fig. 8 shows the p-values across all ten scenarios for the considered IM-EDP pairs. It can be clearly seen that most of the p-values are larger than 0.05. An exception is the CAR,0.4 for Δb, for which none of the scenarios pass the p-value check with respect to the magnitudes of the earthquakes. Therefore, CAR,0.4 is not an optimal FO IM for Δb. Although there are a few p-values smaller than 0.05 for other IM-EDP pairs, they are considered to be acceptable due to their excellent performance in the metric of proficiency, which is a composite measure of

![Fig. 5. Percentage changes of modified dispersion for FO IMs with respect to optimal traditional IMs across all scenarios for demand parameters: (a) maximum column drift ratio, (b) residual column drift ratio, (c) maximum bearing deformation and (d) maximum soil surface displacement.](image)
efficiency and practicality. Sample linear regression of residuals using FO IMs are plotted in Fig. 9, including $SI_{\alpha}$ for $\gamma_{c,max}$ in scenario S3 and $SI_{da}$ for $\Delta_s$ in scenario S7.

5.4. Summary of the performance of the identified optimal fractional order IMs

Gathering all the above results, Table 8 lists the recommended FO IMs with their optimal $\alpha$ values that outperform traditional optimal IMs for different EDPs. Note that the IMs listed in upper positions are those that perform better and should be selected in priority. It is worth noting that the optimality of these FO IMs is limited to the studied extended single pile-shaft-supported bridges in liquefiable and laterally spreading ground.

6. Conclusions

This paper investigates the performance of fractional order seismic intensity measures (FO IMs) for probabilistic seismic demand modeling of extended pile-shaft-supported highway bridges subjected to earthquake-induced liquefaction and lateral spreading in the transverse direction. Simplified soil-foundation-bridge models considering structural and geotechnical material uncertainties as well as geometric parameters are used as case studies to explore recent and newly proposed fractional order IMs for response prediction of structures with complex geotechnical conditions. Common metrics such as efficiency, practicality, proficiency and sufficiency are adopted to quantify the improvements offered by the fractional order intensity measures compared with traditional intensity measures that are normally obtained using integer order calculus. Several fractional order intensity measures

Fig. 6. Variation of optimal $\alpha$ based on the metric of proficiency with respect to different scenarios in terms of demand parameters: (a) maximum column drift ratio, (b) residual column drift ratio, (c) maximum bearing deformation and (d) maximum soil surface displacement.

Fig. 7. Sample probabilistic seismic demand models using FO IMs for: (a) maximum column drift ratio of scenario S0, (b) maximum soil surface displacement of scenario S5.
are studied in this paper, including peak ground fractional response, \( PGR_\alpha \), cumulative absolute fractional response, \( CAR_\alpha \), modified cumulative absolute fractional response, \( CAR_5_\alpha \), and spectrum acceleration at 2.0 s considering a SDF system with fractional damping, \( Sad_{2.0}\alpha \), as well as a conventional SDF system with fractional response, \( Sar_{2.0}\alpha \), and spectrum intensity for a fractionally damped SDF system, \( SId_{\alpha} \), or a conventional SDF system with fractional response, \( SIr_{\alpha} \). Several of these newly proposed FO IMs are introduced by extending common intensity measures. Therefore, these measures reflect important features of ground motions for geotechnical and soil-structure systems, while capturing fractional order features that have been noted to have relation to soil-foundation-structure interaction (SFSI). Engineering demand parameters considered in this study include maximum and residual column drift ratios, denoted by \( \gamma_{c,\text{max}} \) and \( \gamma_{c,\text{res}} \), respectively, maximum bearing deformation, \( \delta_b \), and maximum soil surface displacement, \( \Delta_s \), that accounts for the seismic liquefaction hazard for liquefied and laterally spreading ground.

Among the studied fractional order intensity measures, \( SId_{-2.0} \), \( SIr_{-3.3} \), \( PGR_{-1.3} \), and \( CAR_{-0.4} \) outperform the optimal traditional intensity measures such as \( PGV \) or \( HI \) for probabilistic seismic demand modeling of bridges in liquefiable and laterally spreading ground. These FO IMs (with their respective optimal fractional orders) significantly improve the metrics of practicality, efficiency, and proficiency, as much as 10–20% in general, while maintaining acceptable sufficiency. In particular, the newly proposed \( SId_{-2.0} \) tends to be the optimal fractional order intensity measure for both structural and geotechnical demand parameters. In addition, \( SIr_{-3.3} \) is also an appropriate option for peak structural demand parameters such as \( \gamma_{c,\text{max}} \) and \( \gamma_{c,\text{res}} \), followed by \( PGR_{-1.3} \). Besides, \( CAR_{-0.4} \) is considered to be an alternative prime fractional order intensity measure for residual structural demand parameters such as \( \gamma_{c,\text{res}} \). As to the seismic liquefaction measure, \( \Delta_s \), \( CAR_{-0.7} \) tends to be an alternative choice after \( SId_{-2.0} \).

### Table 8

<table>
<thead>
<tr>
<th>( \gamma_{c,\text{max}} )</th>
<th>( \gamma_{c,\text{res}} )</th>
<th>( \delta_b )</th>
<th>( \Delta_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FO IM</td>
<td>( \alpha ) value</td>
<td>FO IM</td>
<td>( \alpha ) value</td>
</tr>
<tr>
<td>( SL_{\alpha} )</td>
<td>-3.3</td>
<td>( SL_{\alpha} )</td>
<td>-2</td>
</tr>
<tr>
<td>( SL_{d,2.0} )</td>
<td>-2</td>
<td>( SL_{d,2.0} )</td>
<td>-2.7</td>
</tr>
<tr>
<td>( PGR_{\alpha} )</td>
<td>-1.3</td>
<td>( CAR_{\alpha} )</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

Fig. 8. Variation of \( p \)-value across different scenarios in terms of various demand parameters versus: (a) magnitude and (b) source distance.

Fig. 9. Sample linear regression of residuals using FO IMs for: (a) maximum column drift ratio versus magnitude for scenario S3 and (b) maximum soil surface displacement versus source distance for scenario S7.

The fractional spectral accelerations (\(\tilde{S}_a(20a)\) and \(\tilde{S}_a(20e)\)) are considered to be slightly less appropriate based on the metrics considered. This may be partly due to the fact that the analyzed soil-bridge models with various geometric scenarios have a relatively wide range of fundamental periods. Responses of these scenarios may not all be highly correlated with the spectral value at the same discrete period of 2.0 s. Hence, future work could explore alternative periods of interest. Also, the studied soil-bridge systems in the presence of liquefaction and lateral spreading are highly nonlinear under seismic loading. Thus, the initial periods of the soil-bridge systems may not dictate the response due to degradation and softening in structures and soils. Hence periods of the system when it is responding in the nonlinear range may also be considered in future studies.

A noticeable limitation of using fractional order intensity measures in practice is the hazard computability in probabilistic seismic hazard analysis. Although addressing the hazard computability is beyond the scope of this paper, a recent work by the authors has provided one of the first ground motion prediction equations for fractional order intensity measures; hence, availability of probabilistic seismic hazard models for fractional order intensity measures is within practical reach. The superiority of several fractional order intensity measures shown in this study serves to underscore the value of further efforts to conduct probabilistic seismic hazard analyses for fractional order intensity measures. Seismic risk estimates founded upon the use of these novel IMs have the potential to enhance the confidence and accuracy in future seismic risk estimates for bridges in liquefiable soils. In addition, the conclusions motivate further investigation of fractional order intensity measures for different types of structures or infrastructure systems where introduction of fractional order intensity measures may similarly help to improve performance predictions.

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